Monthly Notices of the ROYAL ASTRONOMICAL SOCIETY

Mon. Not. R. Astron. Soc. **402,** 2715–2719 (2010) doi:10.1111/j.1365-2966.2009.16091.x

Strange stars with different quark mass scalings

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Accepted 2009 November 22. Received 2009 November 22; in original form 2009 May 11

ABSTRACT

We investigate the stability of strange quark matter and the properties of the corresponding strange stars for a wide range of quark mass scalings. The calculations show that the resulting maximum mass always lies between $1.5\,\mathrm{M}_\odot$ and $1.8\,\mathrm{M}_\odot$ for all the scalings chosen here. Strange star sequences with a linear scaling support the lowest gravitational masses, and a change (increase or decrease) of the scaling around the linear scaling leads to a higher maximum mass. Radii always decrease with the mass scaling. Thus, the larger the scaling, the faster the star might spin. In addition, the variation of the scaling causes an order of magnitude change of the strong electric field on the quark surface. This field is essential to the support of possible crusts of strange stars against gravity and thus may have some astrophysical implications.

Key words: dense matter – elementary particles – equation of state – stars: interiors.

1 INTRODUCTION

When studying the equation of state (EOS) of ordinary quark matter, the crucial point is to treat quark confinement in an appropriate way. As an alternative to the conventional bag mechanism (in which quarks are asymptotically free within a large bag), one way to obtain confinement is based on the density dependence of the quark mass, and thus the variation of quark mass with density mimics the strong interaction between quarks, which is the basic idea of the quark mass-density-dependent model.

Originally, the interaction part of the quark mass was assumed to be inversely proportional to the density (Fowler, Raha & Weiner 1981; Chakrabarty, Raha & Sinha 1989; Chakrabarty 1991, 1993, 1996), and this linear scaling has been extensively applied to study the properties of strange quark matter (SQM). However, this class of scaling is often criticized because it lacks a convincing derivation (Peng et al. 1999). Subsequently, a cubic scaling was derived based on the in-medium chiral condensates and linear confinement (Peng et al. 1999) and has been widely used since (Lugones & Horvath 2003; Zheng et al. 2004; Peng, Wen & Chen 2006; Wen, Peng & Chen 2007; Peng, Li & Lombardo 2008). This derivation is still not well justified, however, as it considered only the first-order approximation of the chiral condensates in the medium. Incorporating higher orders of the approximation would non-trivially complicate the quark mass formulas (G.X. Peng, private communication). There are, however, other mass scalings in the literature (Dey et al. 1998; Wang 2000; Zhang et al. 2001; Zhang & Su 2002, 2003).

Despite the large uncertainty in the quark mass formulas, the quark mass-density-dependent model is no doubt only a crude approximation to quantum chromodynamics. For example, the model may not account for the quark system in situations where realistic quark vector interaction is non-ignorable. However, we cannot obtain a general idea of how the strong interaction acts from the fundamental theory of strong interactions to hand (i.e. quantum chromodynamics). Until this stimulating controversy is solved, we feel justified in taking the pragmatic point of view and using the model. This work does not claim to explain how Nature works. It may, however, shed some light on what may happen in interesting physical situations. In this respect, the quark mass-density-dependent model has been, and still is, an interesting framework.

The aim of this paper, then, is to clarify to what extent this scaling model is appropriate for studying the properties of SQM. To this end, we treat the quark mass scaling as a free parameter with which to investigate the stability of SQM and the variation in the predicted properties of the corresponding strange stars (SSs) within a wide scaling range. Furthermore, we try to demonstrate the general features of SSs related to astrophysical observations, whatever the value of the free parameters.

The paper is organized as follows. In Section 2 we describe the formalism applied to calculate the EOS of the SQM in the quark mass-density-dependent model. In Section 3 we present the structure of stars made of this matter, including the mass–radius relationship, spin frequency and electric properties of the quark surface. Finally, in Section 4 we present our main conclusions.

2 THE MODEL

As is usually done, we consider SQM as a mixture of interacting u, d and s quarks and electrons, where the mass of the quarks $m_q (q = u, d, s)$ is parametrized with the baryon number density n_b as follows:

$$m_q \equiv m_{q0} + m_\mathrm{I} = m_{q0} + \frac{C}{n_\mathrm{b}^x}, \tag{1}$$

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where C is a parameter to be determined by stability arguments. The density-dependent mass m_q includes two parts: one is the original mass or current mass m_{q0}, and the other is the interacting part $m_{\rm I}$. The exponent of density x, that is, the quark mass scaling, is treated as a free parameter in this paper.

Denoting the Fermi momentum in phase space by $\nu_i (i = u, d, s, e^-)$, the particle number densities can be expressed as

$$n_i = g_i \int \frac{\mathrm{d}^3\mathbf{p}}{(2\pi\hbar)^3} = \frac{g_i}{2\pi^2}\int_0^{\nu_i} p^2\,\mathrm{d}p = \frac{g_i \nu_i^3}{6\pi^2}, \tag{2}$$

and the corresponding energy density as

$$\varepsilon = \sum_i \frac{g_i}{2\pi^2}\int_0^{\nu_i} \sqrt{p^2 + m_i^2}\, p^2\,\mathrm{d}p. \tag{3}$$

The relevant chemical potentials μ_u, μ_d, μ_s and μ_e satisfy the weak-equilibrium condition (we assume that neutrinos leave the system freely):

$$\mu_u + \mu_e = \mu_d,\, \mu_d = \mu_s. \tag{4}$$

For the quark flavour i we have

$$\begin{aligned}\mu_i &= \frac{\mathrm{d}\varepsilon}{\mathrm{d}n_i}|_{\{n_{k\neq i}\}} = \frac{\partial \varepsilon_i}{\partial \nu_i}\frac{\mathrm{d}\nu_i}{\mathrm{d}n_i} + \sum_j \frac{\partial \varepsilon}{\partial m_j}\frac{\partial m_j}{\partial n_i} \\ &= \sqrt{\nu_i^2 + m_i^2} + \sum_j n_j \frac{\partial m_j}{\partial n_i} f\left(\frac{\nu_j}{m_j}\right),\end{aligned} \tag{5}$$

where

$$f(a) \equiv \frac{3}{2a^3}\left[a\sqrt{1+a^2} - \ln\left(a + \sqrt{1+a^2}\right)\right]. \tag{6}$$

It can clearly be seen from equation (5) that, because the quark masses are density-dependent, the derivatives generate an additional term with respect to the free Fermi gas model.

For electrons, we have

$$\mu_e = \sqrt{\left(3\pi^2 n_e\right)^{2/3} + m_e^2}. \tag{7}$$

The pressure is then given by

$$\begin{aligned} P &= -\varepsilon + \sum_i \mu_i n_i \\ &= -\Omega_0 + \sum_{ij} n_i n_j \frac{\partial m_j}{\partial n_i} f\left(\frac{\nu_j}{m_j}\right) \\ &= -\Omega_0 + n_{\rm b}\frac{\mathrm{d}m_{\rm I}}{\mathrm{d}n_{\rm b}} \sum_{j=u,d,s} n_j f\left(\frac{\nu_j}{m_j}\right),\end{aligned} \tag{8}$$

with Ω_0 being the free-particle contribution:

$$\begin{aligned}\Omega_0 = -\sum_i \frac{g_i}{48\pi^2}\Bigg[&\nu_i\sqrt{\nu_i^2 + m_i^2}\left(2\nu_i^2 - 3m_i^2\right) \\ &+ 3m_i^4 \,\mathrm{arcsinh}\left(\frac{\nu_i}{m_i}\right)\Bigg].\end{aligned} \tag{9}$$

The baryon number density and the charge density can be given as

$$n_{\rm b} = \frac{1}{3}(n_u + n_d + n_s), \tag{10}$$

$$Q_{\rm q} = \frac{2}{3}n_u - \frac{1}{3}n_d - \frac{1}{3}n_s - n_e. \tag{11}$$

The charge-neutrality condition requires $Q_{\rm q} = 0$.

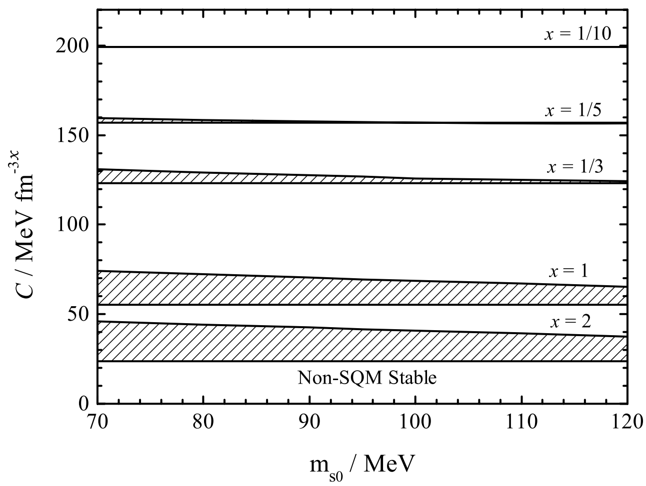

Figure 1. The stability window of the strange quark matter at zero pressure for quark mass scaling parameters of $x = 1/10, 1/5, 1/3, 1, 2$. The stability region (shaded) is where the energy per particle is lower than 930 MeV and two-flavour quark matter is unstable.

Solving equations (4), (10) and (11), we can determine n_u, n_d, n_s and n_e for a given total baryon number density $n_{\rm b}$. The other quantities are obtained straightforwardly.

In the present model, the parameters are: the electron mass $m_e = 0.511$ MeV, the quark current masses m_{u0}, m_{d0} and m_{s0}, the confinement parameter C and the quark mass scaling x. Although the light-quark masses are not without controversy and remain under active investigation, they are anyway very small, and so we simply take $m_{u0} = 5$ MeV, $m_{d0} = 10$ MeV. The current mass of strange quarks is 95 ± 25 MeV according to the latest version of the Particle Data Group (Yao et al. 2006).

We now need to establish the conditions under which the SQM is the true strong-interaction ground state. That is, we must require, at $P = 0$, $E/A \leq M(^{56}\mathrm{Fe})c^2/56 = 930$ MeV for the SQM and $E/A > 930$ MeV for two-flavour quark matter [where $M(^{56}\mathrm{Fe})$ is the mass of ^{56}Fe] in order not to contradict standard nuclear physics. The EOS will describe stable SQM only for a set of values of (C, m_{s0}) satisfying these two conditions, which is given in Fig. 1 as the 'stability window'. Only if the (C, m_{s0}) pair is in a shaded region can SQM be absolutely stable, and therefore the range of C-values is very narrow for a chosen m_{s0}-value. As shown in Fig. 1, the width of the allowed region decreases for decreasing value of x. When $x = 1/10$ it approaches a very narrow band around $C = 199.1$ MeV fm^{-3x}.

In Fig. 2 we illustrate the density dependence of $m_{\rm I}$ with the quark mass scalings $x = 1/10$, $1/3$, 1, 3. The calculation is carried out with $m_{s0} = 95$ MeV and C-values corresponding to the upper boundaries defined in Fig. 1 (the calculations are done with the same parameters hereafter); that is, the system always lies in the same binding state (for each x), namely E/A = 930 MeV. We present those C-values in the bottom row of Table 1. Clearly, the quark mass varies in a very large range from very high-density regions (asymptotic freedom regime) to lower-density regions, where confinement (hadron formation) takes place. The density dependence of $m_{\rm I}$ is compared with Dey et al.'s (1998) scaling (dash–dotted line).

3 RESULTS AND DISCUSSION

The resulting EOSs of SQM are shown in Fig. 3 for all considered models. Because the sound velocity $v = \mid dP/d\rho \mid ^{1/2}$ should be

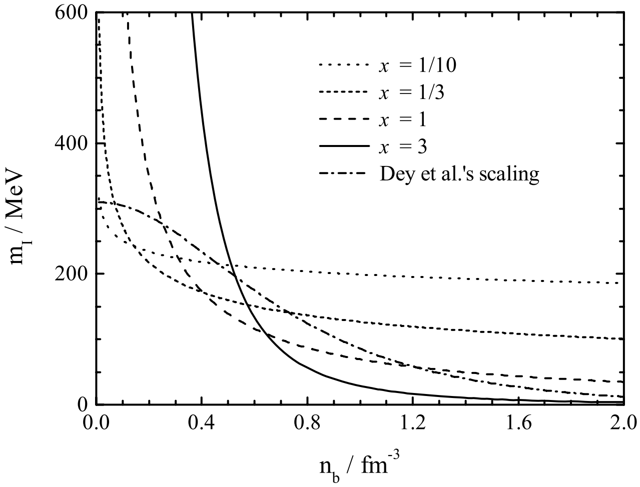

Figure 2. The density dependence of $m_{\rm I}$ for quark mass scaling parameters of $x = 1/10, 1/3, 1, 3$. The calculation is carried out with $m_{s0} = 95\,$MeV and C-values presented in the bottom row of Table 1 (see text for details). Dey et al.'s (1998) scaling (dash–dotted line) is shown for comparison.

Table 1. Calculated results for the gravitational mass, radius, central baryon density (normalized to the saturation density of nuclear matter, $n_0 = 0.17\,\mathrm{fm}^{-3}$) and maximum rotational frequency for the maximum-mass star of each strange star sequence. The calculation is carried out with $m_{s0} = 95\,$MeV and C-values presented in the bottom row of this table.

x	1/10	1/5	1/3	1	2	3
$M/\mathrm{M}_\odot$	1.78	1.66	1.61	1.56	1.61	1.62
R/km	13.2	10.5	9.38	8.10	7.97	7.89
$n_{\rm c}/n_0$	4.35	6.47	7.88	10.1	10.2	10.3
$f_{\max}$/Hz	1066	1446	1691	2072	2159	2194
C/MeV fm^{-3x}	199.1	157.2	126.8	69.5	41.7	28.8

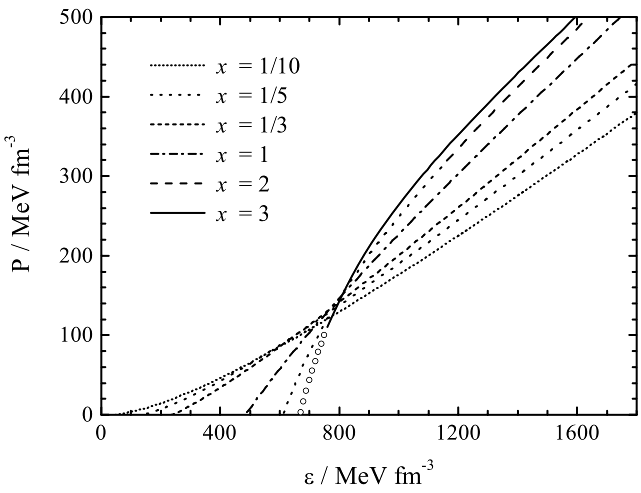

Figure 3. The equations of state of strange quark matter for all considered models. The unphysical region determined by this condition is denoted by circles (see text for details).

smaller than c (the velocity of light), the unphysical region determined by this condition is displayed with circles. For the x-values chosen here, the EOSs have quite different behaviours at low densities, basically falling into two groups. At small scalings ($x = 1/10, 1/5, 1/3$) the pressure increases slowly with density, whereas the curve turns to increase rapidly with density at relatively large

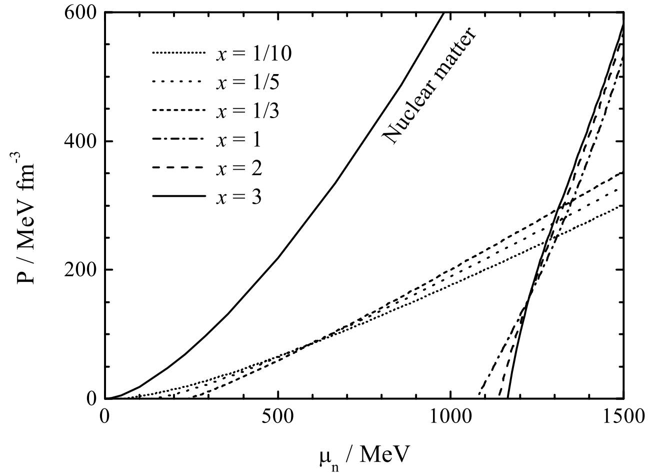

Figure 4. The total pressure as a function of neutron chemical potential in strange quark matter for all considered models, and comparison with that of typical nuclear matter (see text for details).

x-values ($x = 1, 2, 3$). Those EOSs cross at $\varepsilon \sim 800\,\mathrm{MeV\ fm}^{-3}$, and then tend to be asymptotically linear relations at higher densities, and a larger x-value leads to a stiffer EOS. We also check the stability of such quark matter, as some of the EOSs in Fig. 3 ($x = 1, 2, 3$) are rather stiff for small pressures. We present in Fig. 4 the total pressure as a function of neutron chemical potential in quark matter for all considered models, and a comparison with that of typical nuclear matter (obtained from the Brueckner–Hartree–Fock approach of Li et al. 2006). We see clearly from the figure that the quark matter tends to be more stable than nuclear matter for all considered models.

The behaviour of EOSs is mirrored in the prediction of the mass–radius relations of the corresponding SSs, as shown in Fig. 5. For the first group, the maximum mass occurs at a low central density (as shown in Table 1), so a higher maximum mass is obtained owing to a stiffer EOS, and, with increasing x-value, the maximum mass is reduced from $1.78\,\mathrm{M}_\odot$ at $x = 1/10$ down to $1.61\,\mathrm{M}_\odot$ at $x = 1/3$. We observe a slight increase of the maximum mass with x-value for

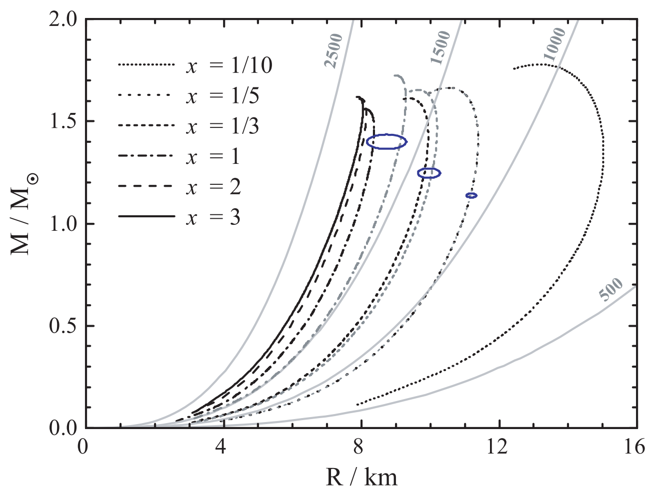

Figure 5. The mass–radius relationships of strange stars for all considered models. $M(R)$ curves for the lower boundaries defined in Fig. 1 with the quark mass scaling parameters $x = 1/5, 1/3, 1$ are shown with grey lines. Contours of the maximum rotation frequencies are given by the light grey curves (Gourgoulhon et al. 1999).

the second group: from $1.56\,\mathrm{M}_{\odot}$ at $x = 1$ up to $1.62\,\mathrm{M}_{\odot}$ at $x = 3$. In any case, the resulting maximum mass lies between $1.5\,\mathrm{M}_{\odot}$ and $1.8\,\mathrm{M}_{\odot}$ for a wide range of x-values (0.1–3), which may be a pleasing feature of this model, namely that it is well controlled. To see the region of stellar parameters allowed by this model, we also plot in Fig. 5 the $M(R)$ curves for the lower boundaries defined in Fig. 1 for $x = 1/5, 1/3$ and 1 (grey lines).

The radii, on the other hand, decrease always with x-value. Employing the empirical formula connecting the maximum rotation frequency with the maximum mass and radius of the static configuration (Gourgoulhon et al. 1999), we obtain the maximum rotational angular frequency $\Omega_{\max}$ as $7730(\mathrm{M}_{\odot}^{\mathrm{stat}}/\mathrm{M}_{\odot})^{1/2}\,(R_{\mathrm{M}_{\odot}}^{\mathrm{stat}}/10\,\mathrm{km})^{-3/2}$ rad s^{-1}. Consequently, a larger x-value results in a larger maximum spin frequency: SSs with $x = 3$ can rotate at a frequency of $f_{\max} = 2194$ Hz. More detailed results can be found in Table 1.

In addition, the surface electric field could be very strong near the bare quark surface of a SS because of the mass difference of the strange quark and the up (or down) quark, which could play an important role in producing the thermal emission of bare strange stars by the Usov mechanism (Usov 1998; Usov 2001). The strong electric field is also crucial to the formation of a possible crust around a SS, and has been investigated extensively by many authors (for a recent development, see Zdunik, Haensel & Gourgoulhon 2001). Furthermore, it should be noted that this electric field may have some important implications for pulsar radio emission mechanisms (Xu, Zhang & Qiao 2001). It is therefore very worthwhile to explore how the mass scaling influences the surface electric field of the stars, and possible related astronomical observations may in turn provide a hint of what the proper mass scaling should be.

Adopting a simple Thomas–Fermi model, one obtains Poisson's equation (Alcock, Farhi & Olinto 1986):

$$\frac{\mathrm{d}^2 V}{\mathrm{d}z^2} = \begin{cases} \frac{4\alpha}{3\pi}\left(V^3 - V_{\mathrm{q}}^3\right) & z \le 0, \\ \frac{4\alpha}{3\pi} V^3 & z > 0, \end{cases} \tag{12}$$

where z is the height above the quark surface, α is the fine-structure constant, and $V_{\mathrm{q}}^3/(3\pi^2\hbar^3 c^3)$ is the quark charge density inside the quark surface. Together with the physical boundary conditions $\{z \to -\infty$: $\mathrm{V} \to V_{\mathrm{q}}$, $\mathrm{d}V/\mathrm{d}z \to 0$; $\mathrm{z} \to +\infty$: $\mathrm{V} \to 0$, $\mathrm{d}V/\mathrm{d}z \to 0\}$, and the continuity of V at $z = 0$ (which requires $V(z = 0) = 3V_{\mathrm{q}}/4$), the solution for $z > 0$ finally leads to

$$V = \frac{3V_{\mathrm{q}}}{\sqrt{\frac{6\alpha}{\pi}}V_{\mathrm{q}}z + 4}\ (\text{for } z > 0). \tag{13}$$

The electron charge density can be calculated as $V^3/(3\pi^2\hbar^3 c^3)$, and therefore the number density of the electrons is

$$n_{\mathrm{e}} = \frac{9V_{\mathrm{q}}^3}{\pi^2\left(\sqrt{\frac{6\alpha}{\pi}}V_{\mathrm{q}}z + 4\right)^3} \tag{14}$$

and, finally, the electric field above the quark surface is

$$E = \sqrt{\frac{2\alpha}{3\pi}}\,\frac{9V_{\mathrm{q}}^2}{\left(\sqrt{\frac{6\alpha}{\pi}}V_{\mathrm{q}}\,z + 4\right)^2}, \tag{15}$$

which is directed outwards.

We see from Fig. 6 (take $x = 1/3$ for example) that, although the electric field near the surface is about 10^{18} V cm^{-1}, the outward electric field decreases very rapidly above the quark surface, and at $z \sim 10^{-8}$ cm the field gets down to $\sim 5 \times 10^{11}$ V cm^{-1}, which is of the order of the rotation-induced electric field for a typical Goldreich–Julian magnetosphere. A change in the mass scaling has two main effects. First, it has a large effect on the surface electric field, and a small scaling parameter leads to an enhanced electric field. The weakening of the electric field would be of almost an order of magnitude (from 10^{17} V cm^{-1} to 10^{18} V cm^{-1}), which may have some effect on astronomical observations. Second, a larger scaling slows the decrease of the electric field above the quark surface.

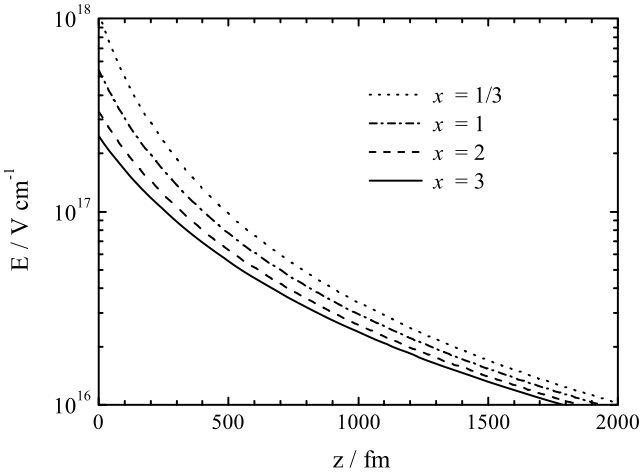

Figure 6. The electric field above the quark surface for quark mass scaling parameters of $x = 1/3, 1, 2$ and 3.

4 CONCLUSIONS

In this paper, we have investigated the stability of SQM within a wide scaling range, namely from 0.1 to 3. We have also studied the properties of SSs made of this matter. The calculations show that the resulting maximum mass always lies between $1.5\,\mathrm{M}_{\odot}$ and $1.8\,\mathrm{M}_{\odot}$ for all the mass scalings chosen here. SS sequences with a linear scaling support the lowest gravitational masses, and a change (increase or decrease) of the scaling parameter around the linear scaling results in a higher maximum mass. Radii always decrease with the mass scaling. Thus, the larger the scaling, the faster the star rotates. In addition, a variation in the scaling may cause an order of magnitude change of the surface electric field, which may have some effect on astronomical observations.

ACKNOWLEDGMENTS

We would like to thank an anonymous referee for valuable comments and suggestions, and we acknowledge Dr. Guang-Xiong Peng for helpful discussions. This work was supported by the National Basic Research Program of China under Grant 2009CB824800, by the National Natural Science Foundation of China under Grants 10778611, 10833002, 10973002, and by the Youth Innovation Foundation of Fujian Province under Grant 2009J05013.

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